**Chapter 4**

**Exercise Set 4.8**

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9.

Factor each number to get:  
27 = 3×3×3  
72= 3×3×2×2×2  
Thus we can find gcd = 3×3 = 9

10.

Factor each number to get:  
5 = 5×1  
9 = 3×3×1  
Thus we can find gcd =1

11.

Factor each number to get:  
7 = 7×1  
21 = 3×7  
Thus we can find gcd = 7

12.

48 = 6\*8 and 54 = 6\*9. So 6|48 and 6|54, and no integer larger than 6 divides both 48 and 54. Hence, the greatest common divisor is 6.

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(a) Factor each number: 12 = 2⋅2⋅3 and 18 = 2⋅3⋅3  
Thus lcm(12,18) = 2⋅2⋅3⋅3 = 36  
(b) Each number have been factorized:  
Thus lcm = 23⋅32⋅5 = 360  
(c) Factor each number: 2800 = 24⋅52⋅7 and 6125 = 53⋅72  
Thus lcm(2800,6125) = 24⋅53⋅72 = 98000

**Exercise Set 5.4**

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1. Let P(n) be the statement to be proved.  
2. Test for n=1,2,3, we have a1=1,a2=3,a3=1+2(3)=7, thus P(1),P(2),P(3) are true.  
3. Suppose it is true for n≤p,(p>3).  
4. For n=p+1, let ap=2k+1,ap−1=2m+1 (k,m are integers), we have ap+1=ap−1+2ap=2m+1+4k+2=2(2k+m+1)+1 which is also odd.  
5. Thus P(p+1) will also be true and we have proved the statement using mathematical induction.

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Description automatically generated

1. Let P(n) be the statement to be proved.  
2. Test for n=1,2,3, we have b1=4,b2=12,b3=4+12=16 all are divisible by 4, thus P(1),P(2),P(3) are true.  
3. Suppose it is true for n≤p,(p>3), that is bp,bp−1... are divisible by 4.  
4. For n=p+1, let bp=4k,bp−1=4m (k,m are integers), we have bp+1=bp−1+bp=4k+4m=4(k+m) which is also divisible by 4..  
5. Thus P(p+1) will also be true and we have proved the statement using mathematical induction.

A math problem with black text

Description automatically generated

1. Let P(n) be the statement to be proved.  
2. Test for n=0,1,2,3, we have c0=2,c1=2,c2=6,c3=3c0=6 all are even, thus P(0),P(1),P(2),P(3) are true.  
3. Suppose it is true for n≤p,(p>3), that is cp,cp−1... are even.  
4. For n=p+1, let cp−2=2m (m is an integer), we have cp+1=3cp−2=6m which is also even..  
5. Thus P(p+1) will also be true and we have proved the statement using mathematical induction.

**Exercise Set 8.3**

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Description automatically generated

a. 17 ≡ 2 (mod 5) <=> 5| 17-2  
True since 5|15 => 15 = 5\*{3} , 3 ∈ Z  
b. 4 ≡ −5 (mod 7) <=> 7 | -9  
false since 9 and 7 are both prime so they don't divide each other  
c. −2 ≡ −8 (mod 3) <=> 3| -8+2  
True since 3|-6 since -6 = 3\*{2), 2∈ Z  
d. −6 ≡ 22 (mod 2) <=> 2|-6-22  
True since 2|-28 since -28 = 2\*{-14), -14∈ Z

**Exercise Set 8.4**

A math problem with numbers

Description automatically generated

a. To verify 3 | (25 - 19 ), we need to check whether the difference between 25 and 19 is divisible by 3.  
25 − 19 = 6 and 3 | 6 since 6 = 3 \* 2.  
Therefore, 3 is indeed a divisor of ( 25−19 )  
So, 3 | ( 25 − 19 ) is true.  
b. 25 ≡ 19 ( mod 3 )  
We have :  
25 = 3 \* 8 + 1 => gcd(25, 3) = 1  
19 = 3 \* 6 + 1 => gcd(19, 3) = 1  
So, both 25 and 19 leave a remainder of 1 when divided by 3, therefore, we can say that 25 and 19 are congruent modulo 3:  
25 ≡ 19 (mod 3)  
c. 25 = 19 + 3k?  
We have :  
25 – 19 = 6 = 3 \* 2  
25 = 19 + 3 \* 2  
So value of k = 2 has the property that 25 = 19 + 3k.

A math equations with black text

Description automatically generated with medium confidence

a. To verify 128 ≡ 2 (mod 7), we can use the following algebraic manipulation :  
128 − 2 = 126 = 7 \* 18 <=> 128 − 2 = 7 \* 18  
which can be rewritten as: 128 ≡ 2 (mod 7)  
  
Also, to verify 61 ≡ 5 (mod 7), we have :  
61 – 5 = 56 = 7 \* 8 <=> 61 – 5 = 7 \* 8  
which can be rewritten as: 61 ≡ 5 ( mod 7 )  
  
b. To verify (128 + 61) ≡ ( 2 + 5) (mod 7), we have :  
128 – 2 = 7 \* 18  
61 – 5 = 7 \* 8  
Therefore, (128 + 61) − ( 2 + 5 ) = ( 128 – 2 ) + ( 61 – 5 )  
= 7 \* 18 + 7 \* 8  
= 7 \* 26  
=> (128 + 61) − ( 2 + 5 ) = 7 \* 26  
which can be rewritten as: (128 + 61) ≡ ( 2 + 5) ( mod 7 )

c. To verify (128 − 61) ≡ ( 2 − 5) (mod 7), we have :  
128 – 2 = 7 \* 18  
−61 – ( – 5 ) = −56 = 7 \* ( −8 )  
Therefore, (128 − 61) − ( 2 − 5 ) = ( 128 – 2 ) + [ −61 – ( – 5 ) ]  
= 7 \* 18 + 7 \*(− 8)  
= 7 \* 10  
=> (128 − 61) − ( 2 − 5 ) = 7 \* 10  
which can be rewritten as: (128 − 61) ≡ ( 2 − 5) ( mod 7 )

d. To verify (128 \* 61) ≡ ( 2 \* 5 ) (mod 7), we have :  
128 \* 61 = 7808 = 7 \* 1115 + 3  
2 \* 5 = 10 = 7 \* 1 + 3  
So, both 128 \* 61 and 2 \* 5 leave a remainder of 3 when divided by 7, therefore, we can  
say that 128 \* 61 and 2 \* 5 are congruent modulo 7.  
Therefore, ( 128 \* 61 ) ≡ ( 2 \* 5 ) ( mod 7 )

e. To verify (1282)≡(22)(mod7), we have :  
1282=(7∗18+2)2=(7∗18)2+2∗(7∗18)∗2+22  
=7∗7∗18∗18+7∗18∗4+4  
=7∗2340+4  
22=(7∗0+2)2=(7∗0)2+2∗(7∗0)∗2+4=4  
So, both 1282 and 22 leave a remainder of 4 when divided by 7, therefore, we can  
say that 1282 and 22 are congruent modulo 7.  
Therefore, (1282)≡(22)(mod7)



26.

First, caculate the gcd( 6664, 765 )  
6664 = 765 \* 8 + 544  
765 = 544\*1 + 221  
544 = 221\*2 + 102  
221 = 102\*2 + 17  
102 = 17 \* 6 + 0  
Therefore, the gcd( 6664, 765 ) = 17  
Then, express 17 as a linear combination of 6664 and 765 using the extended Euclidean algorithm  
17 = 221 – 102\*2  
= 221 – ( 544 – 221 \* 2 ) \* 2  
= 221 – 544 \* 2 + 221\*4  
= 544 \* ( -2 ) + 221\*5  
= 544 \* ( -2 ) + ( 765 - 544\*1 ) \*5  
= 544 \* ( -2 ) + 765 \*5 - 544 \*5  
= 765 \*5 + 544 \* ( -7 )  
= 765 \*5 + ( 6664 - 765 \* 8 ) \* ( - 7 )  
= 6664 \* ( - 7 ) + 765 \* 61  
Therefore, the linear combination of 6664 and 765 is :  
17 = -7(6664) + 61(765)

27.

First, caculate the gcd( 4158, 1568 )  
4158 = 1568 \* 2 + 1022  
1568 = 1022 \* 1 + 546  
1022 = 546 \* 1 + 476  
546 = 476 \* 1 + 70  
476 = 70 \* 6 + 56  
70 = 56 \* 1 + 14  
56 = 14 \* 4 + 0  
Therefore, the gcd( 4158, 1568 ) = 14  
  
Then, express 14 as a linear combination of 4158 and 1568 using the extended Euclidean algorithm :  
14 = 70 \* 1 – 56 \* 1  
= 70 – ( 476 – 70 \* 6 ) \* 1  
= 476 \* (–1) + 70 \* 7  
= 476 \* (–1) + ( 546 – 476 \* 1 ) \* 7  
= 546 \* 7 + 476 \* (–8)  
= 546 \* 7 + ( 1022 – 546 \* 1 ) \* (–8)  
= 1022 \* (–8) + 546 \* 15  
= 1022 \* (–8) + ( 1568 – 1022 \* 1 ) \* 15  
= 1568 \* 15 + 1022 \* (–23)  
= 1568 \* 15 + (4158 – 1568 \* 2 ) \* (–23)  
= 4158 \* (–23) + 1568 \* 61  
  
Therefore, the linear combination of 4158 and 1568 is :  
14 = –23(4158) + 61(1568)